Irregular Geometrical Shapes

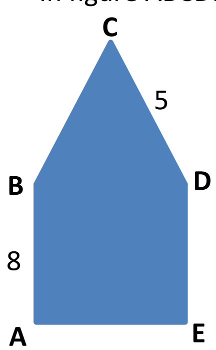
These questions generally ask you to find the perimeter, area or volume of a shape that you have never seen before.

Many students panic because they think that there must be some formula that they are overlooking—not so. For the SAT, you only need to know how to calculate basic measurements of triangles, circles, squares and rectangles, shapes with which you are already quite familiar.

When you get an irregular shape, all you need to do is figure out how to divide it into shapes that you already know. Then perform the required calculations.

Consider the following problem:

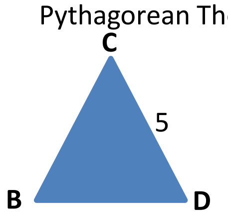
In figure ABCDE, BA = AE, and BC = CD. Find the area of figure ABCDE.



Notice that the figure has five sides. There are no known formulas for calculating the area of a five-sided figure. However, if you look closely, you notice that the figure is really just a triangle juxtaposed with a quadrilateral. You know how to calculate the areas of these two types of figures. And, if you forget, the SAT provides you with these formulas at the beginning of each math section.

Now let's start to analyze the problem. Let's consider the figure, ABDE. We know that BA = AE. Therefore, all sides of the figure must equal 8. This figure is a square. Find the area by taking one of the sides and, as the name of the figure suggests, square it. The area of the square ends up being 64.

Now, let's consider the triangle. The formula for the area of a triangle is $A = \frac{1}{2}bh$. The square and the triangle share a side, BD, which happens to be 8. Thus, b = 8. Now, let's find the height by applying the Pythagorean Theorem. Draw a line from point C to bisect BD into two right triangles.



From the center of BD to D, we have a measure of 4, which we will let be one leg of the right triangle. The height will constitute the other leg. 5, side CD, is the hypotenuse. $a^2 + 4^2 = 5^2$. $a^2 + 16 = 25$. A = 3.

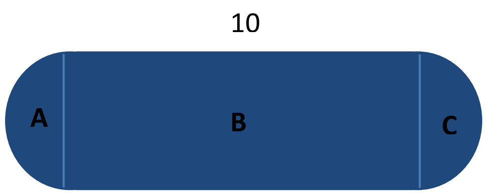
Perhaps you recognized early that we have two 3-4-5 triangles. If so, without applying the Pythagorean Theorem, you would have known that the height is 3.

Now that we have found the height, we can apply the Area formula for triangles, A = ½ bh. The total base is 8 and the height, we just discovered, is 3. We now have A = ½ (8 * 3). A = 12.

Now that we have the area of the square, 64, and the area of the triangle, 12, we only need to add the two together to get the area of the total irregular figure: 64 + 12 = 76.

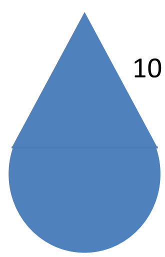
Practice Set:

1.



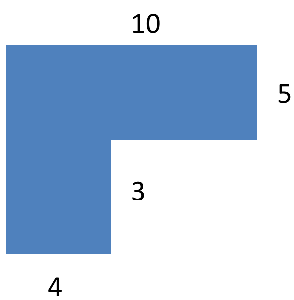
In the above figure, regions A and C are identical semi-circles on either side of rectangular region B, which has an area of 5. Find the area of the entire figure.

2.



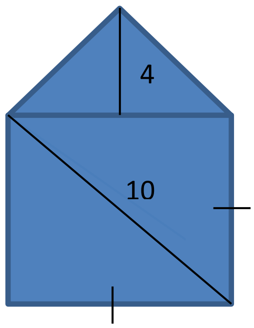
In the above figure, an equilateral triangle is adjoined to a semicircle. Find the area of the figure.

3.



Find the area of the above figure.

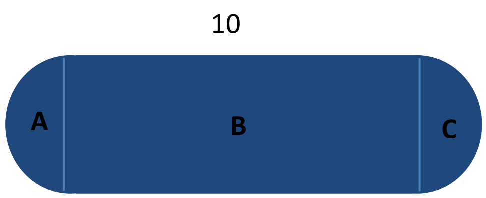
4.



Find the perimeter of the figure above.

Answers Key

1.



In the above figure, regions A and C are identical semi-circles on either side of rectangular region B, which has an area of 5. Find the area of the entire figure.

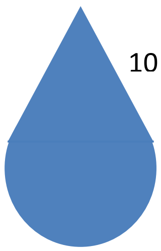
Solution:

Let's start with region B. If the length of the rectangle is 10 and the complete area is 5, then each of the perpendicular lengths must be ½. (A = lw; 5 = 10w; w = ½). Now we can treat sections A and C as one complete circle with a diameter of ½. The area of this circle is indicated by the following formula:

$A = \pi r^2$. The radius is ½ of ½ or ¼. Thus, the area is ¼ π. Now add the area of the rectangle to get the area of the total figure.

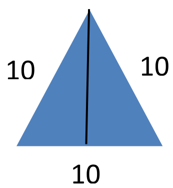
Answer: 5 + ¼ π.

2.



In the above figure, an equilateral triangle is adjoined to a semicircle. Find the area of the figure.

First, let's find the area of the triangular region. A = ½ bh. Since all the sides of the triangle are the same, we know that the base of the triangle is 10. Let's bisect the triangle so that we can find the height. Bisecting the base means that we have two right triangles, each with a base of 5. In this scenario, we can apply Pythagorean Theorem in order to get the height.



$10^2 = 5^2 + h^2$

$100 = 25 + h^2$

$h^2 = 75$

h = 5√3

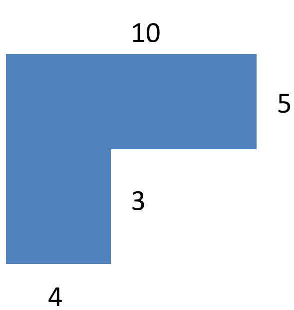
We can now calculate the area of the triangle: A = ½ bh; A = ½ (10) (5√3). A = 25√3.

Now that we have the area of the triangle, we can ind the area of the semicircle, then add the two measurements together. The area of a full circle is: $A = \pi r^2$. A = $\pi 5^2$. A = 25 π. Now divide the entire quantity by 2 to get 12.5 π. We can now add the two ares together to get the area of the total figure:

12.5 π + 25 π

Answer: 37.5 π.

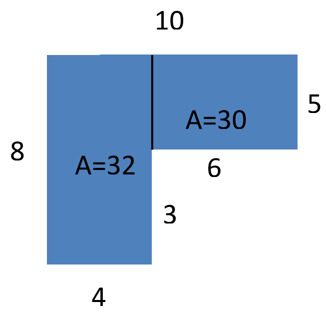
3.



Find the area of the above figure.

Solution:

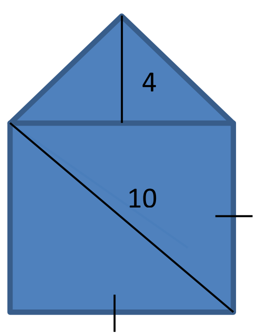
- Divide the figure into rectangles.
- Fill in all measurements that are implied
- Calculate the area of each individual rectangle.
- Finally, add the two areas together



32 + 30

Answer: 62

4.



Find the perimeter of the figure above.

Solution:

Based on the congruency marks on the bottom and right sides of the figure, we can infer that all sides of the lower portion of the figure are equal. Thus, the lower portion is a square.

We can apply our rules of special right triangles to find the length of each side. We have a square bisected diagonally, which means we have a 45-45-90 triangle, with a hypotenuse of 10. We can find the length of each leg with the following formula.

$10 = x\sqrt{2}$. Square both sides to find x. $(10)^2 = (x\sqrt{2})^2$

Once you simplify the equation, $x = 5\sqrt{2}$, which is the length of each side of the square.

Now let's find the lengths of the triangular sides by applying Pythagorean Theorem. The height of 4 already bisects the base, which we now know is $5\sqrt{2}$. Divide this whole thing by 2 to get the measure of the base of each right triangle: $2.5\sqrt{2}$. Now apply the Pythagorean Theorem: $(2.5\sqrt{2})^2 + 4^2 = c^2$

$25/2 + 16 = c^2$

$C = \sqrt{28.5}$. To get the perimeter of the entire figure, multiply one side of the square by 3 (remember that the square and the triangle share a side): $15\sqrt{2}$. Now add the two sides of the triangle (again, we would not add the third side, given that it is shared with the square).

Answer: $15\sqrt{2} + 2\sqrt{28.5}$.